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A Receding Time Horizon Optimal Feedrate Control with Cross-Coupled Structure for Multiaxial Systems

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This work is concerned with the development of digital contouring controller for multiaxial servosystem. Digital optimal contouring controller is proposed to improve the contouring performance by coordinating each of the controllers of multiple feed drives. The optimal control formulation explicitly includes the contour error in the performance index to be minimized. The contouring control is simulated for straight line and circular contours. Substantial improvement in contouring performance is obtained for a range of contouring conditions. Both steady state and transient error measures have been considered. The simulation results show that the proposed controller reduces contouring errors considerably as compared to the conventional uncoupled control for biaxial systems. The presented study has established the potential of the proposed controller to improve contouring performance. The concepts used here seem to be general enough to be useful in other coordinated motions.

Key Words: Receding Time Horizon, Contouring Control, Cross-Coupled Structure

1. Introduction

In multiaxal systems, the motion of each axis must be coordinated to achieve specific performance objectives. Coordinated motion where the motion of two or more axes needs to perform together is essential in many manufacturing processes, for example, machining by CNC machine tools, painting or welding using robots, IC wire bonding, printing, etc. Such objectives call for the design of contouring control systems for multiaxial systems. The contouring control involves decomposition of desired linear or curved contours into reference motion commands to be tracked by each axis and coordinated multiaxial motion controls. The system must be capable of following desired contours accurately by coordinating the motion along each axis.

Many classical and modern control theories have focused on enhancing the performance of individual motion axis. The feedback controller for each axis is designed independently without regard to the motion of other axes. To obtain good contouring accuracy, each axial servomechanism must accurately track the axis command input. However, for multiaxial applications, the individual axis control approach may result in degraded contouring performance for coordinated motion due to such factors as unsymmetrical disturbances, mismatch in axial dynamics, nonlinear trajectories, etc.

Many researchers have studied the motion coordination problem. The motivation mostly came from attempting to improve the contouring performance of machine tool feed drive systems. As for controller design for a single axis, Koren (1983) has provided guidelines for design of a position loop controller based on a first order model for a velocity loop. The position loop gain is recommended to be chosen to minimize the performance criteria of IAE (Integral of Absolute Error) in order to take into account properly the conflicting requirements of low steady state error

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and low overshoot. Bollinger et al.(1980) have noted that the following error for each axis is the most important dynamic error to be considered in contouring applications. They use feedforward compensation to reduce the following error. Doraiswami and Gulliver (1984) designed a robust controller consisted of two dynamic elements, a servocompensator and a stabilizing compensator. A hybrid simulation study with typical operating conditions for different contours shows superior performance of the controller orerv conventionally designed controllers. In Computer Numerical Control machines, a desired path of feed drive is known beforehand. The knowledge has been used profitably for improved control system performance by Tomizuka et al for several applications. Tomizuka and Whitney (1975) presented a solution to the optimal discrete time finite preview problem using stochastic optimal control theory. The preview action has been shown experimentally (Tomizuka and Fung, 1980; Tomizuka, 1989) to result in large reduction in contour error. Tomizuka also developed a Zero Phase Error Tracking Controller (1987), a kind of feedforward control schemes and a repetitive controller (1989, 1991), a sort of learning control schemes, in the framework of a discrete time control. In applications to machine tool control (Tung and Tomizuka, 1994), he has shown that the control schemes enhance the tracking capability by increasing the closed loop system bandwidth and consequently result in improved machining performance through reduction in contour errors.

All the preceding controllers result in independent or uncoupled control for each axis. Several approaches are presented below which couple the multiaxes together for design of the controllers. Sarachik and Ragazzini (1957) employed a master-slave cross-coupled structure between the two axes in a biaxial system. When the magnitude of the position error along one axis gets larger than a certain threshold, the other axis is slowed down. The control implemented by Sarachik et al leads to nonuniform velocity and will result in jerky motion. Koren and Ben Uri (1972) and Koren (1980) have proposed a symmetrical cross-coupled structure for improved contouring performance. A weighted contour error is combined with each of the individual axis errors. It may be noted that this reference is the first work that uses the contour error explicitly in determining the corrective action. Koren and Lo (1991) developed a variable gain cross-coupled controller in which the gain in the contour error controller varied with the contour. Kulkarni and Srinivasan (1985) have also investigated the cross-coupled compensator scheme in detail. They developed different forms of contouring controller applying a corrective control action for each axis on the basis of contour error independently of the individual axis controllers. They also introduced an optimal control formulation which explicity includes the contour error in a performance index (1989). Recently, Chiu and Tomizuka (1994) presented a Lyapunov based controller synthesis approach. They defined a Lyapunov function including tracking error and contour error and constructed a control law guaranteeing exponential decrease of the Lyapunov function through a two step backstepping procedure. Mc Nab and Tsao (1994) defined an optimal control problem with a receding time horizon, which gives the controller some preview information about the upcoming contour and allows the performance of the controller to be determined by the choices of the weighting on tracking error, contour error and control effort.

In this paper, we propose a new discrete time optimal contouring controller for general multiaxal systems. Proportional position loop compensator is designed by conventional techniques independent of the contouring controller since most of commercially available multiaxial systems are already equipped with P-control. The contouring controller provides an additional corrective action to specifically improve the contouring performance. The controller employs full state feedback to minimize a performance index which explicitly weights on quadratic contouring error and control efforta future finite horizon, and therefore combines contour tracking and preview action in a single framework. Since the control concepts developed here are general enough, the controller does not need to be reformulated for use on different contours and will be useful in other applications. Simulation studies demonstrate that the control action designed by using the receding time horizon formulation gives improved contouring performances in spite of a variety of error sources such as mismatched dynamics between axes, disturbances, and nonlinear trajectories in contour tracking.

2. Computation of Contour Error

Figure 1 represents the geometric relationship between the tracking error e and the contour error ε. The tracking error e, is the difference between the commanded position P_r and the actual position P. The contour error ε is defined as the point on the contour nearest the actual position P_c, minus the actual position P. In general, the contour error for curved contours cannot be directly computed during real-time motion. Therefore, a contour error estimate suitable for real-time computation for arbitrary curved contour is required and derived as follows:

Tracking error vector:

$$e = P_r - P \tag{1}$$

Contour error vector:

$$\varepsilon = P_c - P \tag{2}$$

Unit velocity vector:

$$V_r = \frac{\dot{P}_r}{\| \dot{P}_r \|},\ V = \frac{\dot{P}}{\| \dot{P} \|} \tag{3}$$

Average unit velocity vector:

$$\overline{V} = \frac{\dot{P}_r + P}{\| \dot{P}_r + P \|} \tag{4}$$

From Eqs. (1) and (2),

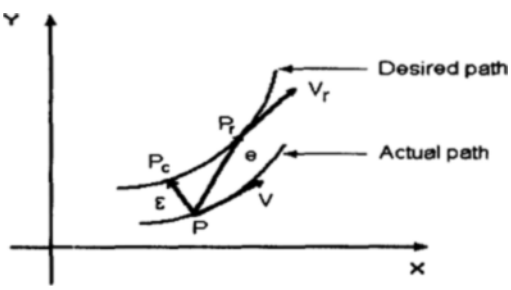

Fig. 1 Tracking and contouring errors on an arbitrary curved contour

$$\varepsilon = P_c - P_r + e \tag{5}$$

Let the transition time from P_c to P_r be denoted as Δt, and the average velocity during that time as V_{ave}, that is

$$V_{ave} = \frac{1}{2}(\dot{P}_r + \dot{P}) \tag{6}$$

Then, the contour error vector can be approximated as

$$\varepsilon = e - (P_r - P_c) \simeq e - V_{ave} \cdot \Delta t \tag{7}$$

Since Δt should be determined such that $\| \varepsilon \|$ is a minimum, solving $\frac{\partial \| \varepsilon \|^2}{\partial(\Delta t)} = 0$ to obtain the expression for Δt and substituting it into Eq. (7) yields a contour error estimate. In two dimensional case, $V_{ave} = [| V_{ave} | \overline{V}_x, | V_{ave} | \overline{V}_y]^T$ and taking the fore-mentioned procedure, ε is simplified as

$$\varepsilon = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} \simeq \begin{bmatrix} V_y(e_x V_y - e_y V_x) \\ -V_x(e_x V_y - e_y V_x) \end{bmatrix} \tag{8}$$

For further approximation of ε, $\overline{V}$ will be replaced by V_r (unit command velocity vector). Then ε will be represented by a combination of reference command inputs and states and it will be used in formulating a performance index for controller design in the next section.

3. Contouring Control

It is easily seen in Fig. 1 that the contour error may be zero even if the tracking error along each

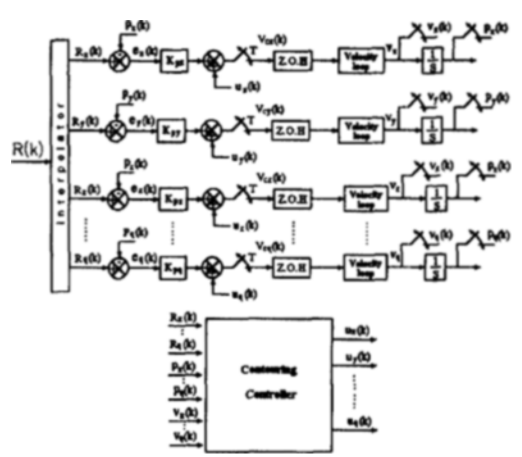

Fig. 2 Block diagram of contouring controller

axis is not. This suggests the possibility that an accurate contour tracking can be achieved through direct involvement of contour error into controller design rather than individual axis performance improvement. This section describes the development of contouring control algorithm for improved contouring performance. A receding time horizon LQ control approach will be applied to enhance contouring accuracy by formulating a penalty function that includes contour error directly and cross-coupled control effort over a finite future time window.

3.1 Contouring control structure

Figure 2 is a block diagram representing the contouring control structure of multiaxial motion control system. A desired path, $R(k)$, is decomposed into individual axis commands, $R_i(k)\,(i=x, y, z, \cdots, q)$ by an interpolator and a position loop comensator for the individual axis constitutes closed loop by proportional controller, K_{pi}, as in the conventional structure. A contouring compensator provides an additional control input, $u_i(k)$, to the closed velocity loop of each axis in order to improve the contouring performance.

In the figure, the input to the velocity loop, Vci $(i=x, y, z, \cdots, q)$ has the following form,

$$V_{ci}(k)=K_{p_i}e_i+u_i(k),\ i=x, y, z, \cdots, q$$
$$=K_{pi}(R_i(k)-p_i(k))+u_i(k) \quad (9)$$

Approximating the closed velocity loop to the first order model as done in the conventional motion control system, the discrete time state equations which give the relation between the control input $V_{ci}(k)$, and the velocity $v_i(k)$, the position $p_i(k)$ are as follows:

$$\begin{bmatrix} p_i(k+1) \\ v_i(k+1) \end{bmatrix}=\begin{bmatrix} 1 & \tau_i(1-e^{-Ts/\tau_i} \\ 0 & e^{-Ts/\tau_i} \end{bmatrix}\begin{bmatrix} p_i(k) \\ v_i(k) \end{bmatrix}$$
$$+\begin{bmatrix} K_{vi}(T_s-\tau_i(1-e^{-Ts/\tau_i}) \\ K_{vi}(1-e^{-Ts/\tau_i}) \end{bmatrix}V_{ci}(k) \quad (10)$$

where τ_i, K_{vi} are the time constant and the steady state gain of the velocity loop, respectively and T_S represents a sampling interval. Putting state vectors of each axis together results in the following state vector and control input vector.

$$x_p(k)=[p_x(k)\,v_x(k)\,p_y(k)\,v_y(k)$$
$$\cdots p_q(k)\,v_q(k)]^T \quad (11)$$
$$V_c(k)=[V_{cx}(k)\,V_{cy}(k)\ \cdots\ V_{cq}(k)]^T \quad (12)$$

From Eqs. (10), (11), (12) and the contour error equation expressed as a combination of states and reference command vectors, the resulting state equation and output equation can be written as

$$x_p(k+1)=A_px_p(k)+B_pV_c(k) \quad (13)$$
$$\varepsilon(k)=C_px_p(k)+C_rR(k)$$

where $R(k)=[R_x(k)\,R_y(k)\ \cdots\ R_q(k)]^T$.

3.2 A Receding time horizon LQ optimal control approach

In this section, a receding time horizon linear quadratic optimal control approach is formulated for multiaxial contour tracking using the state and output equations established in the previous section and a contouring control input vector u (k) is obtained in that framework.

From Eqs. (9), (11), and (12)

$$V_c(k)=-K_px_p+K_cR(k)+u(k) \quad (14)$$

where K_p and K_c are matrices of appropriate dimension whose elements are composed of K_{pi} and $u(k)=[u_x(k)\,u_y(k)\ \cdots\ u_q(k)]^T$. Substituting Eq. (14) into Eq. (13) leads to the following modified state and output equations.

$$x_p(k+1)=(A_p-B_pK_p)x_p(k)+B_pK_cR(k)$$
$$+B_pu(k)$$
$$\varepsilon(k)=C_px_p(k)+C_rR(k) \quad (15)$$

To account for contouring, the performance index to be minimized is of the following form.

$$J=\varepsilon^2(k+N)S_\varepsilon+\sum_{i=k}^{k+N-1}[Q\varepsilon^2(i)$$
$$+u^T(i)Ru(i)] \quad (16)$$

The first term weights the final contour error and the first term in the braket is an explicit measure of the contour error, while the second term weights the square of the contouring control inputs. N represents the length of a preview horizon.

By applying the principle of optimality similarly to the basic finite horizon LQ tracking problem, the optimal input is determined as follows:

$$u(k)=-[B_p^TH_{pp}(k+1)B_p$$

$$+R]^{-1}B_p^T[H_{pp}(k+1)[(A_p-B_pK_p)x_p(k)+B_pK_cR(k)]+H_{pr}(k+1)] \quad (17)$$

where $H_{pp}(k)$ and $H_{pr}(k)$ can be obtained from the following Riccati equations

$$H_{pp}(i)=(A_p-B_pK_p)^TH_{pp}(i+1)\cdot(A_p-B_pK_p)-(A_p-B_pK_p)^TH_{pp}(i+1)B_p\cdot[R+B_p^TH_{pp}(i+1)B_p]^{-1}B_p^T\cdot H_{pp}(i+1)(A_p-B_pK_p)+C_p^TQC_p \quad (18)$$

$$H_{pr}(i)=(A_p-B_pK_p)^T[I-H_{pp}(i+1)B_p\cdot[R+B_p^TH_{pp}(i+1)B_p]^{-1}\cdot B_p^TH_{pr}(i+1)+[C_p^TQC_r+(A_p-B_pK_p)^TH_{pp}(i+1)\cdot[I-B_p[R+B_p^TH_{pp}(i+1)B_p]^{-1}\cdot B_p^TH_{pp}(i+1)]B_pK_c]R(i) \quad (19)$$

with terminal conditions

$$H_{pp(k+N)}=C_p^TS_\epsilon C_p \quad (20)$$

$$H_{pr(k+N)}=C_p^TS_\epsilon C_pR(k+N) \quad (21)$$

As seen in the above equations, the optimal control input is calculated at each time step given the contour trajectory over the preview window N.

4. Contouring Control for Biaxial System

A biaxial motion control system is considered in this section to validate the developed contouring control algorithm. The command trajectories used are straight line contours and circular contours since arbitrary contours are typically approximated by straight lines and circular segments.

4.1 Dynamic model for biaxial system

The design of control algorithm and its performance evaluation require the model of a plant. In this work, dynamic models are obtained theoretically using experimental data for feed drives of CNC machining center equipped with high speed $X-Y$ table. Each feed drive system of the $X-Y$ table includes a 3-phase AC servomotor, a 5mm/pitch ball screw, anti- friction roller bearing slide, and a tachometer and an encoder for motor shaft velocity and position feedback, respectively.

The feed drive servo for each axis uses an Yaskawa velocity servopack that provides digital commutation for the AC motor, PWM type current amplification, analog proportional plus integral (PI) velocity feedback control. Position sensing is provided by a 1500pulses/rev optical shaft encoder. The pulse signal is multiplied 4 times after decoding and sent to a pulse counter. The signal from the counter is finally read by a Data Acquisition Board each sampling interval. Then, the linear resolution of the X-Y table is 1/1200mm.

The closed velocity loop was excited by a 1.0 V step input as a test input for a velocity command. The input and output signals were recorded at a sampling frequency of 5000 Hz for 1024 samples. The response is shown in Fig. 3. It is seen that the response has higher order dynamic characteristics. For simplicity, the system is modeled approximately as a second order lag. The parameters of the second order model are obtained from the experimental data. The continuous time model derived as below has the natural frequency ω_n the damping ratio ζ, and the steady state gain k_v at 6389.5(rad/sec), 0.5731, and 4.5, respectively.

$$G_x(s)=G_y(s)\cong\frac{k_v w_w^2}{s^2+2\zeta\omega_n s+\omega_n^2} \quad (22)$$

Then, the above second order model is further approximated to the following first order model.

$$G_x(s)=G_y(s)\simeq\frac{k_v}{\tau_v s+1} \quad (23)$$

where τ_v is 0.0003.

Figure 3 compares the responses of the experimental system and its approximated second and first order models. The first order model is then converted to the equivalent discrete-time version using Eq. (10).

4.2 Contouring performance evaluation

The effectiveness of the contouring controllers proposed above is studied by means of digital simulations of the dynamics of the drives and the controllers. A sampling interval of 1.0 millisecond is used for implementation of the digital controllers. The feed drive dynamics of Eq. (22) are integrated employing a 4th order Runge-Kutta algorithm with a time step of 0.2 msec. The contouring control is exercised for straight line and circular contours. Both contours are implemented at the same traversing speed of 30m/min. The responses of the contouring control and the independent axis control are compared for the straight line contours in Figs. 4 and 5. In Figs. 4 (a), and (b), where the feed drive systems of both X and Y axes have matched dynamics, both systems result in zero steady-state error while, in Figs. 5(a), (b) with the two axes having 50% mismatched dynamics, the errors of the contouring control are reduced to less than one tenth of those of the independent axes control.

Figs. 6(a), (b), and (c) show the tracking responses for the circular contours of radius of 50mm by the two control schemes when both X and Y feed drive systems have matched dynamics. The initial position of the feed drive is on the commanded trajectory. The resulting variations of the contour errors are plotted in Fig. 6(c). The independent axis control has considerable magnitude of contour error even with the matched axis dynamics as shown in Fig. 6(a), whereas the contouring control improves the corresponding

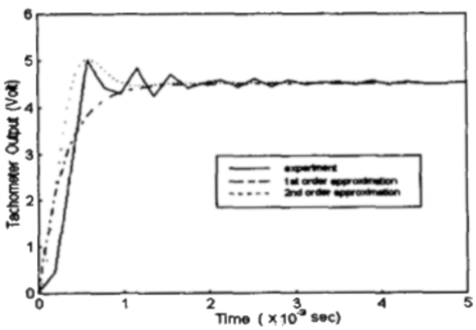

Fig. 3 Experimental velocity loop response to 1 volt step input of vref and its approximated model responses

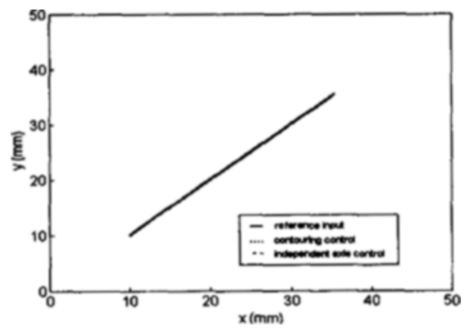

Fig. 4(a) Straight line contour response for independent axis control and contouring control with matched dynamics

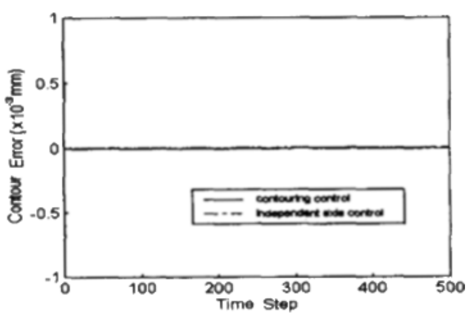

Fig. 4(b) Straight line contour error response for independent axis control and contouring control with matched dynamics

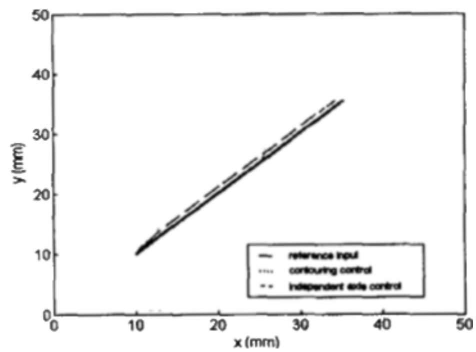

Fig. 5(a) Straight line contour response for independent axis control and contouring control with mismatched dynamics

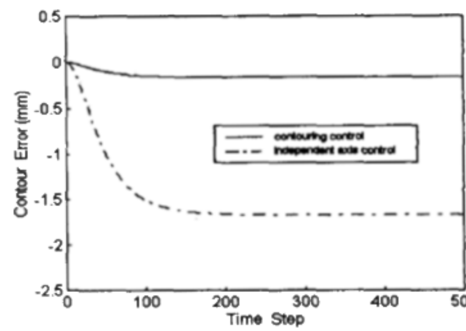

Fig. 5(b) Straight line contour error response for independent axis control and contouring control with mismatched dynamics error measures remarkably as seen in Figs. 6(b), and (c).

The same circular contour was implemented with simulations of the two control schemes when the X and Y feed drive dynamics has 50% mismatched parameter values. The resulting behaviors are plotted in Figs. 7(a), (b), and (c). As seen in Fig. 7(a), the independent axes control reponse has an elliptically distorted contour shape due to the mismatched dynamics. However, the commanded circular shape is recovered through the improved contouring performance of the contour-

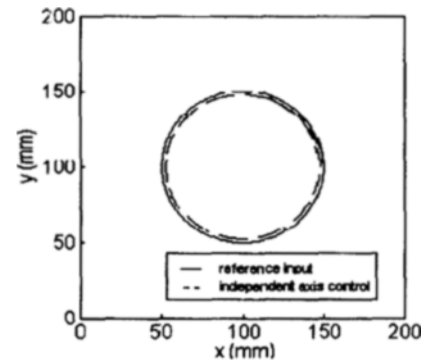

Fig. 6(a) Circular contour response for independent axis control with matched dynamics

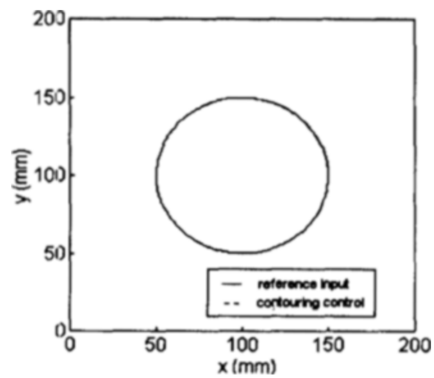

Fig. 6(b) Circular contour response for contouring control with matched dynamics

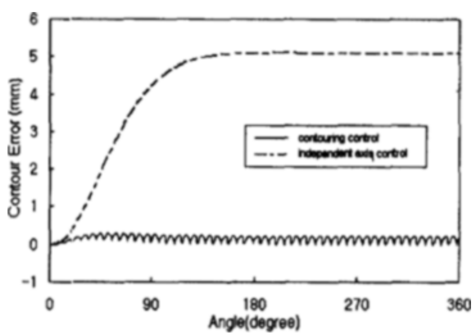

Fig. 6(c) Circular contour error responses for independent axis control and contouring control with matched dynamics

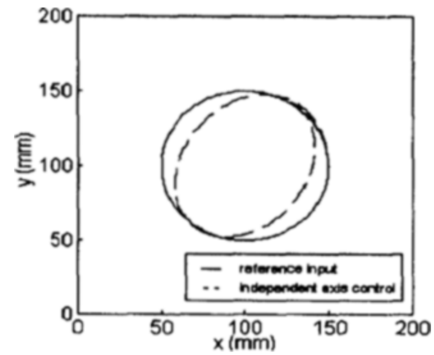

Fig. 7(a) Circular contour response for independent axis control with mismatched dynamics

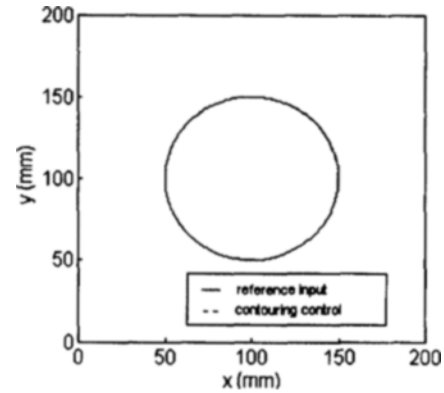

Fig. 7(b) Circular contour response for contouring control with mismatched dynamics

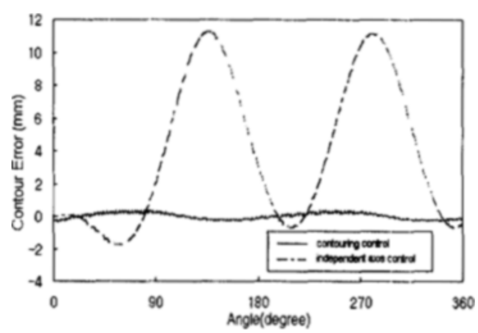

Fig. 7(c) Circular contour error responses for independent axis control and contouring control with mismatched dynamics ing controller in Fig. 7(b). We see that the contouring controller enhances the contouring performance by about a factor of ten over the independent axes controller in Fig. 7(c).

In a receding time horizon optimal control formulation, the main issues are the selection of an appropriate index of performance, weighting factors, and a preview length involved. Figure 8 shows the simulated responses of the circular contour error for various weighting factors S_ε, and Q. The figure shows that the increase of the weighting factors on state variables results in lower radial errors as expected. However, as shown in the Fig. 8, larger values in S_ε, and Q result in more oscillatory behavior of the contour error. In fact, an appropriate choice of the weighting factors should involve compromise between increase in accuracy and speed of response and reduction in damping.

5. Conclusion and Recommendation for Further Work

In order to enhance the contouring performance for multiaxial motion, an additive correctional action is introduced. The receding time horizon LQ controller has been developed and applied by formulating the performance index that includes the contour error and the control effort, over a finite future time window. The developed controller was evaluated by simulation based on low order dynamic models of the feed drives in a biaxial motion system. Extensive evaluation for straight line and circular contours has shown their effectiveness in reducing contour errors as compared to the independent axes controller. The extent of improvement is much greater when the dynamics of the feed drives are not matched. The design parameters needed to adjust performance are limited to relative weight between the contour error and the control effort, and the preview length. The results show that more weighting on the contour error results in smaller contour errors.

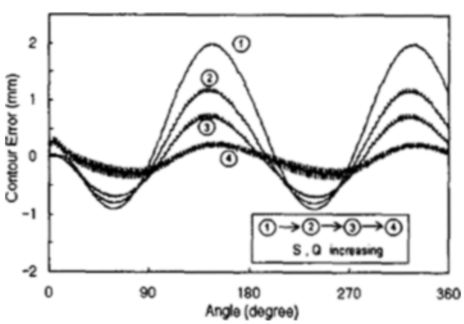

Fig. 8 Circular contour error responses for different weighting values

The contouring trajectories considered here involve biaxial motion. Extensions of the developed controllers for contouring motion in three-dimensional space and for different two-dimensional contours such as parabolic contours are a logical next step. Also, the performance evaluation through detailed experimental work should be conducted. There are many extensions which can be applied to the receding horizon LQ approach employed in this work. The index of performance could incorporate integral and/or differential control actions by introducing augmented state equations and weighting matrices.

The control concepts developed here are general enough to be useful in many applications. Those are coordinate measurement machines, multiple degree-of-freedom robots, chemical process control that needs to coordinate the variables of temperature, flow rate and pressure, etc. The direct approach to maintain coordination of the outputs of the different control loops should be useful in all those applications.

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